

A Research on the Effects of Using 3D Dynamic Geometry Software as Learning Material of the Cross-Section of a Cube for Junior High School Students

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Abstract

Since 2022, one ICT device per student has been available at junior high schools in Japan. However, the use of ICT in mathematics education in Japan is still low. In this research, we asked first-year junior high school students to consider the cutting plane of a cube using 3D dynamic geometry software in the teaching of spatial figures. We confirmed that the students' knowledge and skills of the cubic cutting plane improved when they operated the 3D dynamic geometry software independently while consulting with each other. On the other hand, regarding students' development of "thinking, judgment, and expression skills" we found that the students were visually misled at the stage of drawing 3-D figures on a flat surface.

1 Introduction

In the research of figures in the mathematics department of junior high schools in Japan, 3-D figures are mainly taught in the first grade of junior high school. The Courses of Study for Junior High Schools published in 2017 demands students to acquire some aims. It defines the aim of knowledge and skills as understanding the positional relationship of lines and planes in space. Also, it defines the aim of thinking ability as finding the nature of 3-D figures and expressing them on the plane by themselves. ([7], p.78)

1.1 Mathematics instruction in a one-student-one-unit environment

The Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT) initially planned to achieve a goal of one ICT device per student by 2023. Due to the Covid pandemic, the learning environment that promotes achieving this goal was established at all junior high schools nationwide rapidly. ([8], p.4). However, ICT has not been utilized in mathematics classes in Japan.

The first supplementary budget for FY2009 and other measures increased the penetration rate of electronic blackboards in public schools, and "there is an undeniable sense that the improvement of mathematics teachers' instruction using ICT has shifted from the effective use of digital teaching materials such as learning materials and software to the improvement of instruction through the use

of electronic blackboards as presentation devices.” ([18], p.2), Yasuno (2019) points out. Even in an environment in which each student has one terminal, the use of ICT by mathematics teachers is still not active.

Tani and Yanagimoto (2017) note that ”in Japanese secondary education, there are very few situations in which ICT is used in textbooks, and the types of ICT handled are very few. ([15], pp.18-19). Although, in Japan, practical research on mathematics education using graphing calculators has been conducted since the 1990s. Onishi (2015) points out that ”Japan is one of the world’s leading countries in the production of ICT equipment, but the reality is that ICT equipment is rarely utilized in mathematics classes.”([12], pp.77-78)

However, by 2022, a learning environment where each student has one ICT terminal has been established nationwide. It is about time mathematics teachers in Japan utilized ICT as a material that makes students learn mathematics effectively. Until 2000, it had been hard for students to understand some figures especially 3D figures, by using ICT applications. After 2010, ICT function has improved, and 3D figures are operated easily on ICT. Therefore, this research examines the effectiveness of using ICT, focusing on the instruction of 3D figures.

1.2 Considerations for teaching 3D geometry and the use of ICT

Kinoshita et al. (2020) analyzed the solution strategies of learners who solve 3-D figure tasks using three media (real objects, tablets, and paper), not only through behavioral observation, but also through experiments measuring brain activity and eye movement. As one of the results, they point out that ”we need to be very careful about teaching methods that easily use tablet materials as a substitute for real materials” ([4], p.96). However, Kinoshita et al. (2020) did not clarify what kind of tablet materials were used in their research.

Aoki et al. (2021) clarified the process of learners’ problem solving when performing a cube cutting task by measuring their eye movement. As one of the results, they revealed that ”the wrong answers show difficulty in identifying a point that is not visible” ([1], 2021, p.128). From this point of view, ”it is important to combine the enhancement of opportunities for manipulative activities using 3D teaching materials and 3D teaching materials using tablets, etc., depending on the situation” ([1], pp.128-129).

Yamada and Tsukamoto (2012) noted that by utilizing ICT in the teaching of cube floor plans, ”the students were able to break free from the biased viewpoint that they had been able to see only from a certain direction by understanding that the viewpoint could be moved by the computer.” ([16], p.43) and ”the activity to confirm the change of viewpoint by computer was effective” ([16], p.43). However, they do not mention instruction regarding the cutting plane of the cube.

Iijima (2021) states that ”when the subject is considered to be the students or a group of students, learning in which ICT is positioned as a tool for activating inquiry and dialogue should be given equal or greater importance than passive learning” ([2],p.22). author think that the aim is to encourage dialogue with others through the use of ICT and to have students explore the nature of mathematics.

Shimizu and Kakihana (1999) stated that teaching students to draw their own diagrams using Cabri, a graphic learning software, ”is an effective method that leads to independent learning.” ([14], pp.56-58). This is a research of drawing instruction of plane figures, but it does not mention whether the use of ICT in teaching 3-D figures also fosters students’ independent learning attitudes.

Juandi et al. (2021) analyzed the effectiveness of Dynamic Geometry Software (hereinafter referred to as the DGS) by reviewing various research papers on mathematics learning. As a result,

Juandi et al. (2021) state "The results of the analysis reveal that the use of DGS has a high positive impact on students' math abilities." ([3], p.29). In mathematics education in Japan, the use of ICT to develop lessons that investigate mathematics is still not very active, but it is clear that mathematics learning using DGS is an approach that is needed even from an international perspective.

Marasabessy and Helsa (2024) claim "This present study has estimated that the utilization of GeoGebra software in geometry lesson provides positive strong effect toward the cultivation of spatial visualization of students. Moreover, it can be justified that significantly GeoGebra-assisted geometry lesson cultivates students' spatial visualization skills." ([6], pp.11-12). However, if the use of GeoGebra can help to improve spatial visualization capabilities, we believe that it can also be used in the consideration of cubic cut surfaces.

2 Purpose of the research

In Japanese mathematics textbooks, there is a lesson called "B Figures". It aims for first-year junior high school students to understand the positional relationship of lines and planes in space and to calculate the surface area and volume of a solid object, (see [9], p.4). Since 1998, the treatment of the cutting surfaces of cube was removed.

2.1 Purpose of this research

In the Courses of Study published in 2017, the content for the first grade lists "knowing the positional relationship of lines and planes in space" in terms of knowledge and skills as one of the matters to be acquired through mathematical activities related to spatial figures ([7], p.78). As part of this "knowing the positional relationships of lines and planes in space", students are required to "know the positional relationships of lines and planes in space". we focused on considering the cutting plane of a cube. Cutting cubes is not covered in the 2017 Courses of Study. Although it was once considered too difficult to handle and was reduced in the past, the author thought that it would be possible to study the cutting plane of a cube using ICT.

In this research, ICT is used in the mathematics classroom to encourage students to intuitively grasp numbers and perform exploratory activities related to the cutting plane of a cube. Specifically, first-year junior high school students are taught about cubic cutting planes using DGS. ICT is used instead of relying on manipulative activities with 3D models when thinking about cubic sectional planes. If the learning effect can be demonstrated in a class using GeoGebra Classic Ver.6.0718(hereinafter referred to as GeoGebra) as a 3D-DGS, ICT can be used in mathematics education in Japan. If we can demonstrate the learning effects of teaching using 3D-DGS, we can show the necessity of using ICT in mathematics education in Japan for further exploration and consideration of mathematics. We believe that the importance of this research can be demonstrated.

Before conducting this activity, we will conduct a pre-survey to assess students' understanding of the section plane of a cube. Then, a post-survey will be administered after the class has practiced using 3D-DGS. In the post-survey, students will be asked to work on problems similar to those in the pre-survey to see if their understanding of cubic cutting planes has deepened. The results of the pre-survey and post-survey are compared quantitatively. The purpose of this comparison is to see if students' understanding of the cubic section plane is improved by having them manipulate the 3D-DGS independently. In addition, we will qualitatively examine whether there was a change in

students' attitudes toward learning about the cubic section plane, based on students' presentations through the mathematical activities and their impressions after the class.

3 Research Methods

This research was conducted from late September to early October 2023 for approximately two weeks, five hours per week. The school where the research was conducted mainly used a non-examined textbook, "Systematic Mathematics 1 Geometry" (hereafter referred to as "Systematic Mathematics"). In this practice, the latter two hours were used for worksheets (hereinafter referred to as "WS").

3.1 Target students

The target students were a total of 77 students in two first-year junior high school classes at a private integrated girls' junior and senior high school in Tokyo, Japan. Both classes were organized so that there would be no bias in academic ability based on the results of an entrance examination. When students enroll in the school, their parents or guardians give written consent for them to participate in the recording and presentation of their learning activities. Therefore, there are no ethical problems.

Each year, an iPad is purchased for each student as they enter their first year of junior high school. In the year 2023 students who entered the school purchased an Apple iPad, 9th Generation Wi-Fi model A2602. The iPads are kitted out with the necessary applications installed and configured for easy use within the school. Each student who will be using the dynamic geometry software also owns one of these iPads. Microsoft Teams (hereinafter referred to as "Teams") and GeoGebra are installed on them. In this practice, we will use GeoGebra as the 3D dynamic geometry software. In addition, author will use Teams to share files of teaching materials and to encourage students to respond to questionnaires.

3.2 Class Flow

In the class, "Systematic Mathematics" was used. Chapter 2, "Spatial Figures," was taught in the following order: Section 1, "Various solids," Section 2, "Planes and lines in space," and Section 3, "Various ways of looking at solids. Section 3 deals with the cross-section of a cube. In the text of the textbook, it is only described in half of a page.

Before beginning the instruction on the cross-section of a cube, the students are asked to answer six questions on the cross-section of a cube in a quiz. This was a pre-survey, and the students had about 10 minutes to answer the questions. The teacher then gave explanations and had the students grade their own answers, and collected them. In the WS, author shared the GeoGebra file (Figure 1) prepared by the author in advance on Teams, and each student downloaded the file and manipulated it on her own terminal to examine it. After the WS instruction, a quiz (6 similar questions about the cross-section of a cube) was given again. This was a post-survey, and the students had about 10 minutes to answer the questions. The teacher gave explanations and allowed the students to grade themselves, and then collected the answers. Finally, the students were asked to give their impressions of the class on cubic sections. The students were asked to fill in their impressions by answering a questionnaire on Teams during class time.

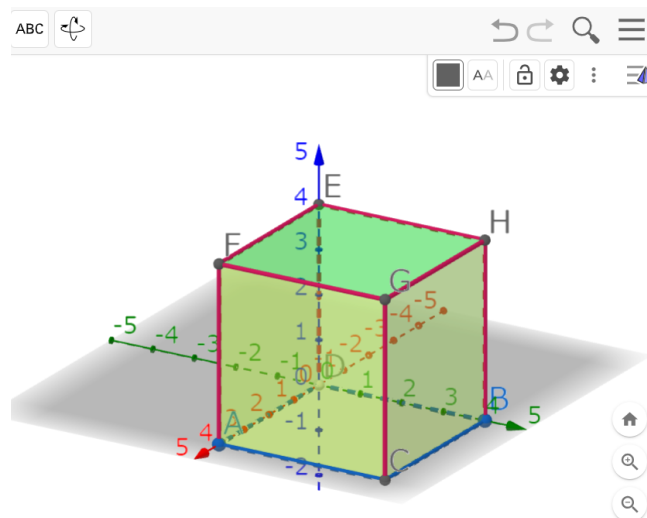


Figure 1: Cube shared in Teams

3.3 Methods of Verification

The purpose of this research is to verify the objectives of Chapter 2 by comparing the results of the pre-survey and the post-survey. We will quantitatively confirm whether the number of correct answers in the pre- and post-survey quizzes increases. In addition, author will analyze the questionnaires administered after the exercise. The questionnaire was developed based on the one used in a previous research by Shimizu and Kakihana (1999), which asked for responses to 1 to 12 items using a five-item method. Students' self-evaluations of "independent learning attitude," "thinking, judging, and expressing," and "knowledge and skills" were examined. By examining the average of the students' self-evaluations, author can verify their "independent learning attitude", "thinking, judgment and expression", and "knowledge and skills".

At the end of the questionnaire, the students were asked, "Please tell me what you think was good and what you regret about studying the cubic section." The results of the free-response section of the questionnaire, as well as the statements made by the students during the lessons using the WS, will be qualitatively verified. The GeoGebra files submitted by the students will also be referenced where appropriate.

The questionnaire was distributed through Forms, an application on Teams, with the questions shown in the attachment at the end of this document, and participants were instructed to answer the questions individually.

4 The worksheet in practice

In this practice, after teaching the aforementioned Chapter 2 of "Systematic Mathematics" in the order of Sections 1 and 2, we prepared and conducted a workshop on the cross-section of a cube in Section 3.

In the pre- and post-survey questions and the WS questions, the students were asked to specify the three points through which the cut plane passes, based on the cube treated in the previous research by Aoki et al. A research by Aoki et al. (2021) revealed that "misrespondents show difficulty in

S₁ : (moves GeoGebra to change the viewpoint of the cross-section)

T : The lengths of the sides extending from PQ and R are equal.

4.2 Activities with GeoGebra (Part 2)

The students were asked to investigate [Q2] and [Q3] using the 3D application feature of GeoGebra. They consulted with other students and taught each other during their investigations. We aimed for the kind of learning that activates inquiry and dialogue as claimed by Iijima (2021) by considering the student group as the main subject and using ICT as a tool. However, we were not able to confirm [Q2] and [Q3] in one class period, so we decided to continue the work in the next class period.

The following is an exchange with a student who shared the screen of [Q2].

T : What shape is this cross-section?

S₂ : It is a hexagon.

T : What kind of hexagon is it?

S₂ : (Adjust the position of points P, Q, and R to the midpoints of the sides and change the viewpoint to a position that is easier to see)

T : It looks like a regular hexagon. The visible edges on each side are the same length. Can you transform it into (2)?

S₂ : (The position of P does not seem to move well. The student will open another file and show her classmates.)

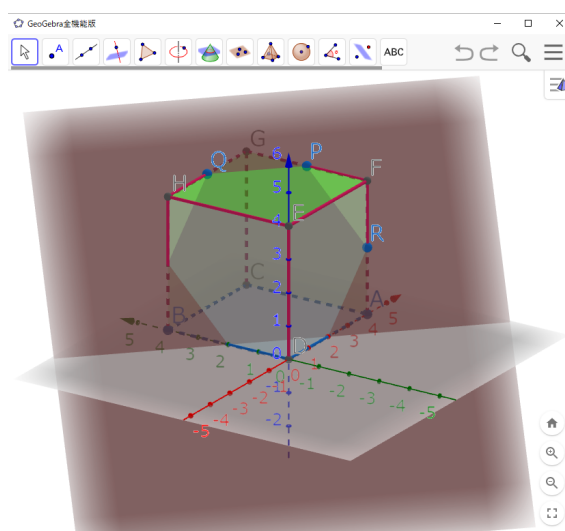


Figure 3: Student's work file for [Q2] (1)

The teacher also shared the students' screenshots of [Q3] and reviewed them, and a post-assessment quiz was given at the end of the second period. The teacher also explained the post-survey quiz to the students and had them grade their own answers and then collected them. Both the pre-survey and post-survey quizzes were administered without the use of ICT. The schools in which author conducted the tests did not require the use of ICT, so author did not use ICT for the quizzes in this practice either.

5 RESULTS

This section reports the results of the pre- and post-survey questions, as well as the classroom situation and the results of the questionnaire.

5.1 Pre and Post Survey Results

The pre-survey and post-survey questions are listed in the Appendix at the end of this report. "Cutting planes of a cube (Part 1)" is the pre-survey question and "Cutting planes of a cube (Part 2)" is the post-survey question. Each survey consists of six questions. Each question asks the participants to draw a cutting plane through the three points P, Q, and R on the edge of the cube (hereinafter referred to as the Fig.) and to name the shape of the cutting plane (hereinafter referred to as the C-S). The author thought that questions (1) to (5) would be improved in the later part of the test, since the tendency of the questions is the same in the pre- and post-tests. Question (6) was asked to see if it could be applied to the pre- and post-tests since the tendency of this question was different between the pre- and post-tests. Of the 77 students, 73 students who took both the pre- and post-tests were analyzed. Scoring was based on a binary score of 1 correct or 0 incorrect, with no subscores. The percentage of correct answers for each of these questions is shown in Figure 4. Table 2 shows the 2×2 crosstabulations and odds ratios of correct and incorrect answers for each question, pre- and post-test. The numbers in the "Number of correct/incorrect answers" column indicate the number of respondents, and the percentages are shown in parentheses. Although the total number of samples exceeded 50, many of the samples in each cell were less than 50, so Fisher's exact test was performed in R. 4.1.2.([13]) The coefficient of correlation (V) was also examined as an effect size. The table 1 of Yanagawa (2023) was used as the index for determining Cramer's coefficient of correlation. ([17], p.126).

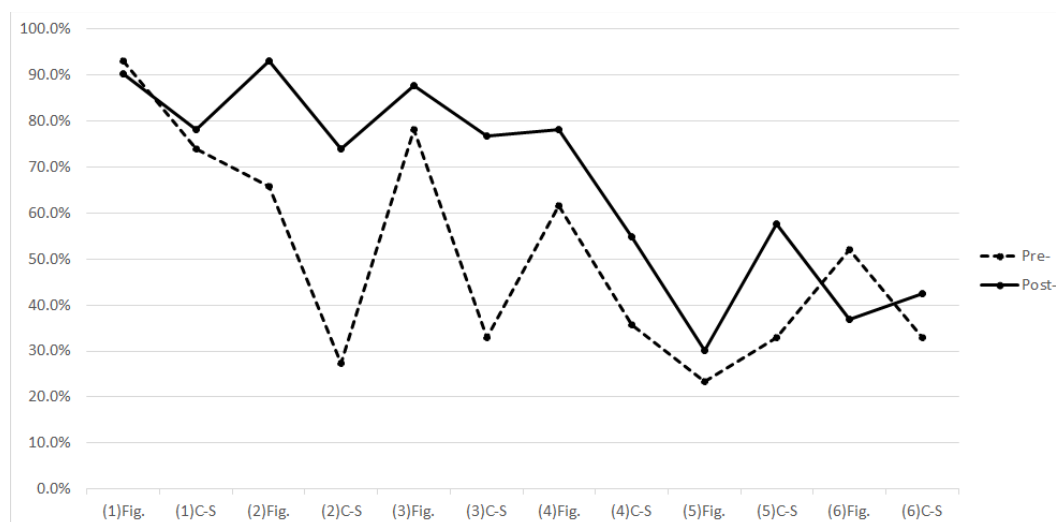


Figure 4: Scoring results of pre- and post-survey questions

Table 1: Index for determining Cramer's linkage coefficients

Cramer's V	Strength of a linkage
0~0.1	Very weak
0.1~0.25	Weak
0.25~0.5	Medium
0.5~1.0	Strong

Table 2: Crosstabulation Tables and Odds Ratios

Question	pre-responses		post-responses		Odds ratio	Odds 95% CI
	Correct	Incorrect	Correct	Incorrect		
(1)Fig.	68(93.2%)	5(6.8%)	66(90.4%)	7(9.6%)	1.442	[0.436 , 4.773]
(1)C-S	54(74.0%)	19(26.0%)	57(78.1%)	16(21.9%)	0.798	[0.372 , 1.709]
(2)Fig.	48(65.8%)	25(34.2%)	68(93.2%)	5(6.8%)	0.141	[0.050 , 0.395]
(2)C-S	20(27.4%)	53(72.6%)	54(74.0%)	19(26.0%)	0.133	[0.372 , 1.709]
(3)Fig.	57(78.1%)	16(21.9%)	64(87.7%)	9(12.3%)	0.501	[0.205 , 1.221]
(3)C-S	24(32.9%)	49(67.1%)	56(76.7%)	17(23.3%)	0.149	[0.072 , 0.309]
(4)Fig.	45(61.6%)	28(38.4%)	57(78.1%)	16(21.9%)	0.451	[0.218 , 0.934]
(4)C-S	26(35.6%)	47(64.4%)	40(54.8%)	33(45.2%)	0.456	[0.235 , 0.887]
(5)Fig.	17(23.3%)	56(76.7%)	22(30.1%)	51(69.9%)	0.704	[0.336 , 1.472]
(5)C-S	24(32.9%)	49(67.1%)	42(57.5%)	31(42.5%)	0.362	[0.184 , 0.709]
(6)Fig.	38(52.1%)	35(47.9%)	27(37.0%)	46(63.0%)	1.850	[0.955 , 3.582]
(6)C-S	24(32.9%)	49(67.1%)	31(42.5%)	42(57.5%)	0.664	[0.338 , 1.302]

Table 3: Results of Fisher's exact test

Question	Cramer's V	p-value	95% CI
(1)Fig.	.050	.765	[0.372 , 6.053]
(1)C-S	.048	.699	[0.345 , 1.831]
(2)Fig.	.339	< .01**	[0.040 , 0.416]
(2)C-S	.466	< .01**	[0.060 , 0.293]
(3)Fig.	.127	.187	[0.181 , 1.321]
(3)C-S	.440	< .01**	[0.067 , 0.327]
(4)Fig.	.179	.047*	[0.202 , 0.989]
(4)C-S	.193	.030*	[0.222 , 0.934]
(5)Fig.	.077	.455	[0.313 , 1.567]
(5)C-S	.248	.005**	[0.174 , 0.747]
(6)Fig.	.152	.096	[0.908 , 3.780]
(6)C-S	.099	.306	[0.320 , 1.373]

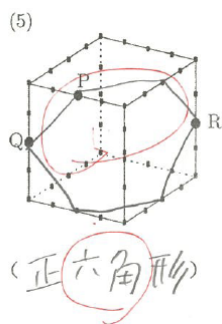


Figure 5: Example 1 of the incorrect answers in the post-survey (5)

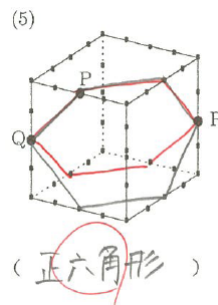


Figure 6: Example 2 of the incorrect answers in the post-survey (5)

Table 4: Number of correct and incorrect answers for (5) figure

	pre-	post-
correct answer	17(23.3%)	22(30.1%)
wrong answer 1	18(24.7%)	23(31.5%)
wrong answer 2	38(52.1%)	28(38.4%)
計	73	73

(2)Fig. shows the effect size ($V = .339$), which is moderately correlated. The odds ratio was 0.141 (95% CI [0.050, 0.395]), and the Fisher's test showed a significant difference ($p < .01$). Similar results were found for (2)C-S and (3)C-S. Although the effect sizes (.179, .193, .248, respectively) were weak for (4)Fig, (4)C-S, and (5)C-S, the odds ratios were 0.451 (95% CI [0.218, 0.934]), 0.456 (95% CI [0.235, 0.887]), and 0.362 (95% CI [0.184, 0.183 CI [0.184, 0.709]]), respectively, and the Fisher's test showed a significant difference ($p < .05$).

As for (5), a significant difference ($p < .05$) in the percentage of correct responses was observed for the (5)C-S, but no difference was observed for the (5)Fig., and even in the post-survey, such incorrect responses as in Figure 5 and Figure 6 were noticeable. The student in Figure 5 did not even notice her error when she scored herself and gave the correct answer. The table 4 summarizes the results of the responses in (5)Fig. The numbers in brackets indicate the number of respondents, and the percentages are given in brackets. Incorrect answer 1" in the table refers to the incorrect answers such as "Figure 5" and "Figure 6". "Incorrect answer 2" represents other incorrect answers.

5.2 Comparing Worksheet Practice with Pre and Post Surveys

Problems like the pre- and post-survey problem (5) are addressed in WS Q2(1), as introduced in the 4.2 section. The fact that the cross-section is a regular hexagon was confirmed by the fact that the lines on each side are diagonals of a right-angled isosceles triangle. Correct answers were checked by moving the viewpoint while sharing the work files of the correct answers (Figure 3). The students who were able to confirm that the cross-section was a regular hexagon using GeoGebra, as shown in Figure 3, were 38 out of 77 students, or about half.

When the students in Figure 5 were asked to consider Question 2(1) of the WS, they were also able to check the cross-section in the file, as in Figure 7, and sketched the correct cross-section. However, in the post-test question, she wrote "Wrong Answer 1". Thus, there were several students who sketched "Wrong Answer 1" on the post-question, even though they were able to check the correct cross-section in the file. In the post-survey question, even though they were able to check the correct cross-section of WS Q2(1) using GeoGebra. This was the case for 9 out of 73 students.

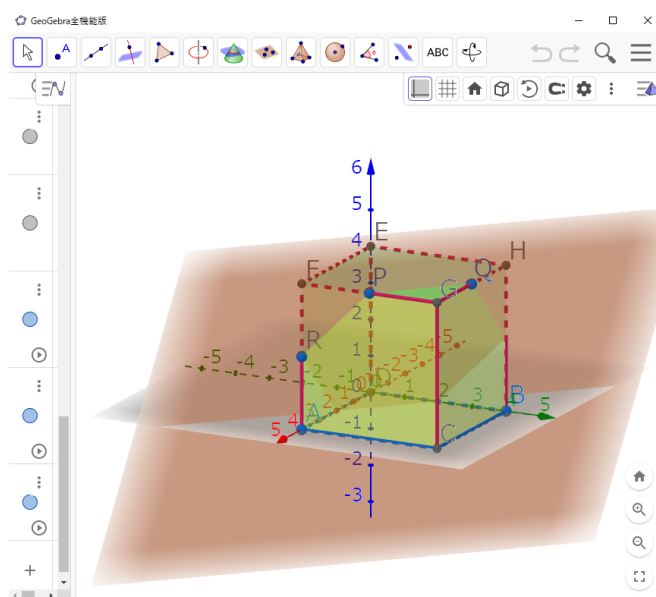


Figure 7: WS Q2(1)

5.3 Results of the Questionnaire

The questionnaire was administered to 77 target students in the form of a questionnaire with their names written on it and answered from Forms. The response period was from October 2 to October 11, 2023, after the second period of this practice. 68 students responded (93.2% response rate) within this period.

5.3.1 Multiple Choice Questionnaire Results

In the multiple-choice questionnaire, students self-rated their attitudes toward "independent learning," "thinking, judging, and expressing," and "knowledge and skills" on a 5-point scale from 1 to 5, where the higher the number, the more they self-rated that they had achieved the goals of the class. The average rating for each item is shown in Table 5. The correspondence between the perspectives of "independent learning attitude," "thinking, judging, and expressing," and "knowledge and skills" and the question items is shown in Table 6. The results of the questionnaire are shown in Figure 8

Table 5: Means for each question

No.	Mean of evaluation	No.	Mean of evaluation
1	4.29	7	4.50
2	4.12	8	4.24
3	4.25	9	4.65
4	4.44	10	4.44
5	2.63	11	4.47
6	4.40	12	4.01

Table 6: Correspondence between perspectives and questions

Perspective	Item No.	Mean
independent learning	1,4,5,6,11,12	4.04
thinking, judgment, and expression	7,8,9	4.46
knowledge and skills	2,3,10	4.27

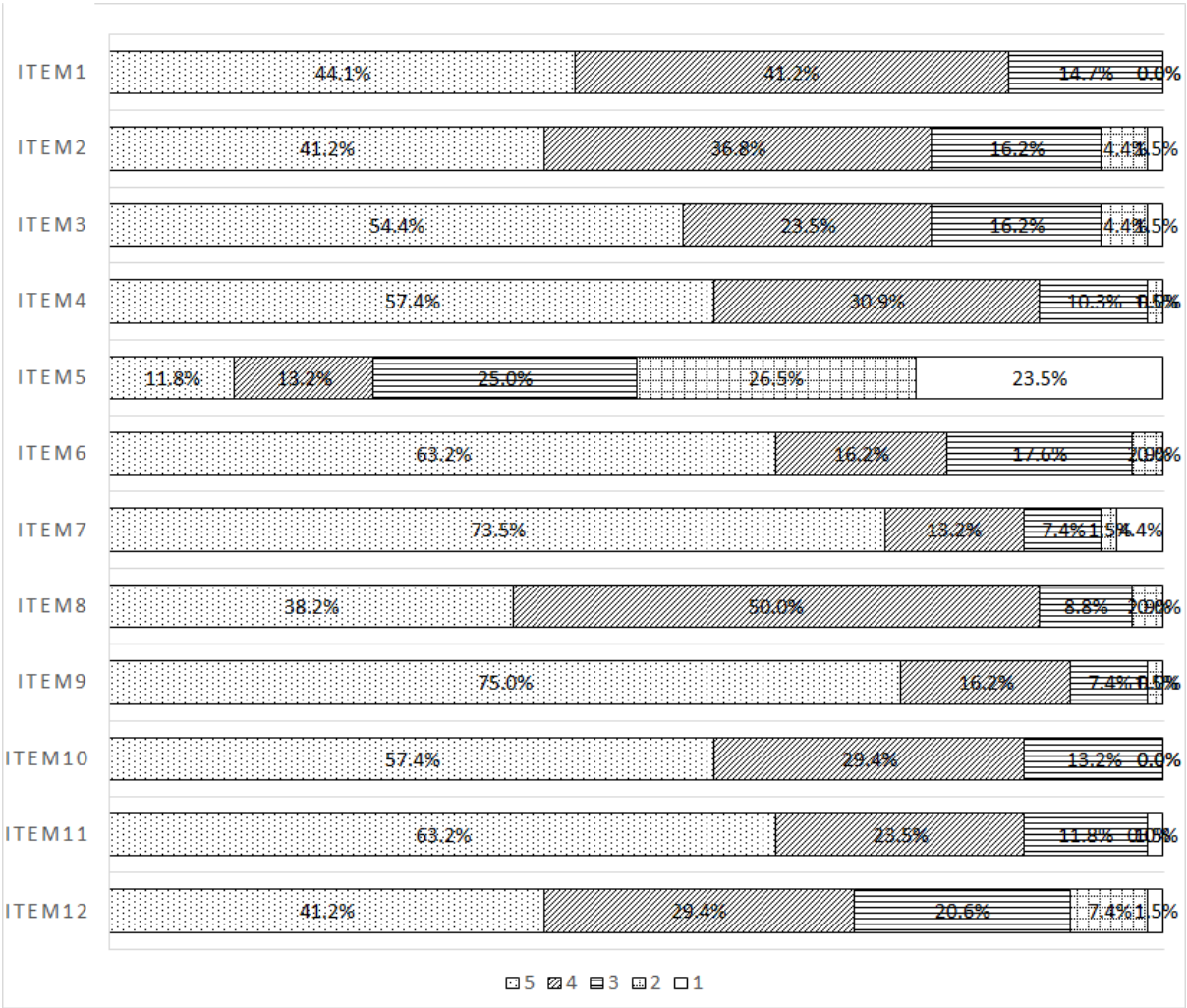


Figure 8: Results of the multiple-choice questionnaire

5.3.2 Results of Descriptive Questionnaire

Question 13 asked students to write free-response answers to the question, "Please tell me what you think was good and what you regret about the study of cubic section planes." The results were analyzed using R ver. 4.1.2. ([13]) with the RMeCab package ver. 1.07 ([10]). 68 responses were analyzed using morphological analysis to break them down into word units, and the following four nouns were noted as frequently used by the author. The noun "GeoGebra" was used in 40 cases, "cutting plane" in 24 cases, "understanding" in 19 cases, and "friend" in 17 cases. The most frequently used adjectives were "easy" (29), "good" (19), and "fun" (7). As verbs, "can" and "understand" were used in 47 and 23 cases, respectively. This analysis was conducted using Kobayashi's (2020) function for creating a KWIC concordance ([5], pp.123-124). Based on these words that we focused on, the following are examples of how students may have collaborated with others and considered them through a trial-and-error process.

- I had a lot of fun learning by thinking creatively with my friends, and I gained a deeper understanding of cross-sections and was very happy when I completed the figure.
- I was able to understand the diagrams while looking at the printouts, but I couldn't present them, so I will have the courage to do so next time.
- I was able to concentrate on the work in cooperation with my friends. Even if I didn't know how to do spatial graphics, I could understand by listening to my friends' presentations and we were able to teach each other.

In addition, several impressions were mentioned that showed an improvement in students' motivation to learn through ICT-supported instruction. The important ones are listed below.

- It was easier to understand when I practiced with GeoGebra than when I explained orally, so I would like to continue to use GeoGebra for problems involving figures.
- It was fun to use GeoGebra for cross-sectional drawings, as if I was doing crafts. I also wanted to review it by myself because I was interested in it.
- It was convenient to use GeoGebra because I could easily see the cut surface. I would like to try many other things.
- GeoGebra was easy to solve because I could see the figure from various angles. However, I could not fully remember how to operate it, and I had to ask my friend many times.

Finally, several students commented that their understanding of the cross-section of a cube was deepened. The important comments are listed below.

- I have always had a hard time understanding 3-D cutting. However, thanks to the teacher's clear explanations, I was able to understand it. I was able to deepen my understanding further by studying with GeoGebra.
- This time, I was not good at cutting cross-sections, but I was able to deepen my understanding by listening to my friends and the teacher.

- I could not see the actual cube from many angles just from the textbook, so GeoGebra allowed me to see it from above and below, and I could color the cross-sectional shapes so I could learn more about the shapes of the cut surfaces.
- Using the iPad, I was able to move the figure freely and see the cut surface, which was something I couldn't understand just by writing it in my notebook, so it became very easy to imagine.

6 Considerations

6.1 Discussion of the Pre and Post Surveys

Half of the 12 questions showed a learning effect in 6 of the 12 questions. Based on the results of the 5.1 section, it is assumed that the learning effect was observed in the understanding of (2)Fig. and (2)C-S. The same is true for (3)C-S, (4)Fig. It can be considered that the 3D-DGS is useful for improving students' understanding of the cross-section of a cube.

The following is a discussion of the questions for which no significant differences were found. The percentage of correct answers to question (1) in Figure 4 indicates that both the pre and post difficulty levels of Figure 4 and C-S were similarly superior to each other. (3)Fig., the only difference was the orientation of the cut surface. Therefore, author do not believe that there was any difference between the pre and post test. Question (6) was an applied problem with a different trend between pre and post, and therefore there was no significant difference in the percentage of correct answers.

The percentage of correct answers in (5)Fig. increased after the test. However, the number of incorrect answers, as in Figure 5 and Figure 6, is about the same as the number of correct answers, as shown in the Table 4. This may be due to students being visually confused. To solve this problem, it is possible to consider teaching materials other than ICT. For example, it may be necessary to consider using a 3D printer to produce 3D model materials.

6.2 Considerations from the Questionnaire

The results of the multiple-choice questionnaire in Table 5 show that the average score for all items is 4.00 or higher, except for question 5. The answer to question 5, "Were you willing to make a presentation in class about the drawing?", which asked about "independent learning attitude", had a mean of 2.63, which was lower than the other items. Therefore, the mean of the overall rating of "independent learning attitude" in the table was 4.04, which was slightly lower than the other two perspectives.

From the comments in the descriptive questionnaire in the comments in section 5.3.2 such as "Listening to my friends' presentations was very good because I could understand them and we could teach each other" and "I felt a sense of accomplishment by discussing opinions with my friends and creating a cross-section" indicate that the use of ICT, as pointed out by Iijima (2021), promoted dialogue with others and allowed students to explore the properties related to mathematics. author believe that the use of ICT, as pointed out by Iijima (2021), promoted dialog with others and allowed students to explore the nature of mathematics. In addition, there were some reflections on question 5, "Did you try to present your drawings willingly in class?". Students were encouraged to explore the nature of mathematics through the use of ICT. The students' comments such as "I would like to have

the courage to make a presentation in the next class” indicate that they would like to improve their attitudes towards learning in the future.

The comments in section 5.3.2 also show an increase in students’ motivation to use ICT in their future studies. The comments in the descriptive questionnaires show that students are actively using ICT to think through trial and error and to develop new ways of expressing themselves.

The word ”understanding” was used positively in 19 of the comments in the descriptive questionnaire, including the one presented here. Comments such as ”Studying with GeoGebra helped me deepen my understanding” and ”Being able to move the figure freely and see the section plane made it much easier for me to imagine” were representative of these comments. The students’ impressions also confirmed the reason for the improvement in the percentage of correct answers in the post-survey questions.

7 CONCLUSIONS

This research was conducted by having students independently manipulate the 3D-DGS to investigate cubic cutting planes. The goal was to confirm that students’ understanding of and attitudes toward learning about cubic sectional planes improved as a result of this activity.

7.1 Effects of Using the 3D-DGS

The results of the pre- and post-surveys (2) to (4) show that the number of correct answers increased significantly for 6 out of 12 questions (Table 3), which was half of the total. This result quantitatively confirms that the learning activity using 3D-DGS by WS was effective in deepening students’ understanding of the cubic section plane. While there was no difference between the pre- and post-assessment for simple questions such as (1), students were able to develop the ability to identify points located in invisible positions as in questions (2) to (4).

In question (5), there was no significant difference in the increase in the number of correct answers. The reason for this is that, as discussed in section 5.2, there were several participants who gave incorrect answers, such as Figure5 and Figure6, even though they could identify the correct cross-section in 3D-DGS, as shown in Figure7 (see Table 4). These errors may be due to the fact that students were visually misled at the stage of drawing the 3-dimensional figure on the plane, even though they were able to correctly observe the cross-sectional view using ICT. Aoki et al. (2021) pointed out that ”the enhancement of opportunities for manipulative activities using 3D learning materials and the combination of 3D learning materials using tablets, etc., depending on the situation” ([1], p.129) should also be considered. In the future, we would like to consider developing 3D learning materials using 3D printers, etc., which have recently become more convenient to use.

7.2 Teaching Practice and Students’ Impressions

In the WS, students exchanged opinions with each other using the 3D-DGS and changed the view-points and color of the cross-sections through a trial-and-error process. The use of ICT, as pointed out by Iijima (2021), encouraged dialog with others and allowed students to explore the nature of mathematics. Several student comments such as ”I was able to deepen my understanding” and ”It became easier to imagine the cutting plane” were also noted. The results of the pre- and post-surveys

confirm that the students themselves felt that their understanding of the cutting plane of the cube had improved.

7.3 Conclusion

The use of ICT to examine the cross-section of a cube in the study of spatial geometry was found to improve students' understanding, and the use of ICT in mathematical activities was found to be effective in stimulating students to interact with others and to explore the properties of spatial geometry. In addition to the quantitative improvement in performance on the survey questions, several students indicated that their understanding had deepened. Changes in motivation to learn were also observed, such as "I am more interested" and "I want to try other things". The importance of using ICT as a tool for exploring mathematics was confirmed in the research of the cross-section of a cube in the study of spatial figures. It can be said that ICT is a useful tool to deepen students' understanding and improve their motivation to learn. Therefore, ICT should be more actively incorporated into the daily teaching of mathematics in Japan as a tool for exploring and thinking about mathematics on a daily basis.

While these effects were observed, some problems were found in the activity of drawing the cross-section confirmed by ICT on a plane, suggesting that not only ICT but also other activities such as showing real models are necessary. In the future, author would like to consider the use of 3D materials using a 3D printer. In addition, since the target students were limited to first-year junior high school students, author would like to consider conducting the survey for high school students in the future.

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Appendix 1

GeoGebra Cubic Cutting Plane Questionnaire

① Were you interested in learning more about cubic cut surfaces using GeoGebra?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

② Did you learn anything new about cubic cut surfaces through your observations with GeoGebra?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

③ Did you learn anything new through this GeoGebra study?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

④ Do you think you were self-motivated to learn about cubic cut surfaces?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

⑤ Did you willingly give a presentation in class about cubic cutting planes?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

⑥ Did you listen attentively to your friends' ideas and opinions about the cutting plane of a cube?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

⑦ Did you discuss the solution of the problem of the cutting plane of a cube with your friends?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

⑧ Were you able to think in different ways about the cutting plane of a cube?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

⑨ Did you have enough time to think about solving the problem of a cubic cutting plane?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

⑩ Was your teacher's explanation easy to understand?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

⑪ Did you enjoy this lesson on cubic sections?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

⑫ Did you want to learn more about cubic figures?

Yes					No
-----+-----+-----+-----					
☐	☐	☐	☐	☐	☐

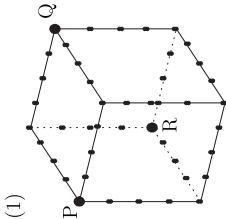
⑬ Please tell me what you thought was good and what you regret about learning about cubic section surfaces.

Appendix 2

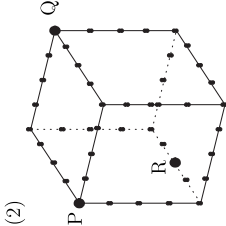
Cutting plane of a cube (Part 1)

Your Name ()

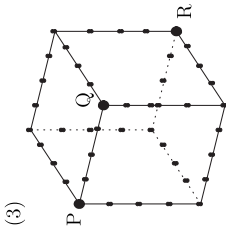
[Question] Draw the shape of the cut surface of the cube below when it is cut in the plane passing through the three points P, Q, and R. Note the symbols in the drawing to indicate which lines are parallel and which lines are equal in length. Also, name the shape of the cut surface in parentheses below.

(1)

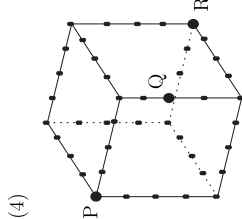
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(2)

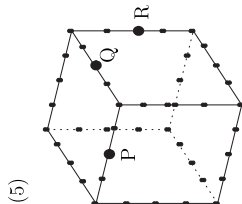
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(3)

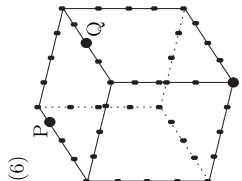
() () ()

(4)

() () ()

(5)

() () ()

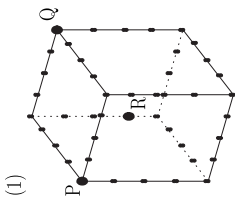
(6)

() () ()

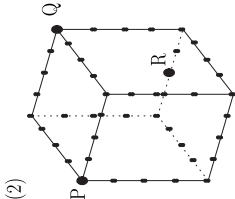
Cutting planes of a cube (Part 2)

Your Name ()

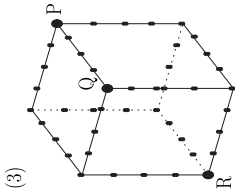
[Question] Draw the shape of the cut (cutting plane) of the cube below when it is cut by a plane passing through the three points P, Q, and R. In the drawing, draw symbols to indicate which lines are parallel and which lines are equal in length. Also, name the shape of the cut surface in parentheses below.

(1)

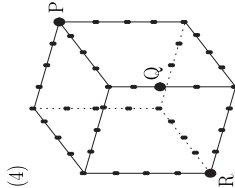
() () ()

(2)

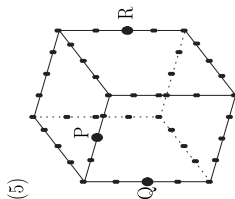
() () ()

(3)

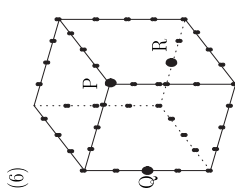
() () ()

(4)

() () ()

(5)

() () ()

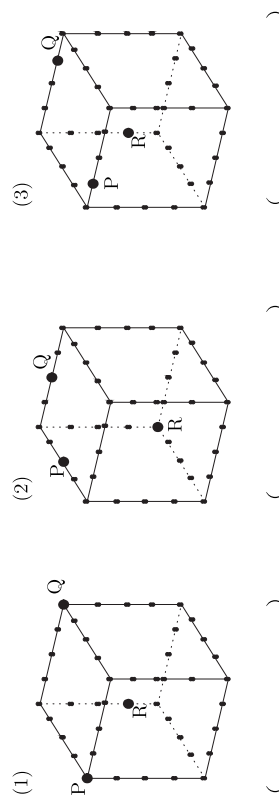
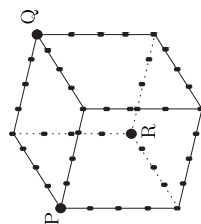
(6)

() () ()

Observing the cut surface of a cube with GeoGebra

[Q1] Let's examine the cut plane in the figure on the right using GeoGebra. Follow the steps below to operate GeoGebra.

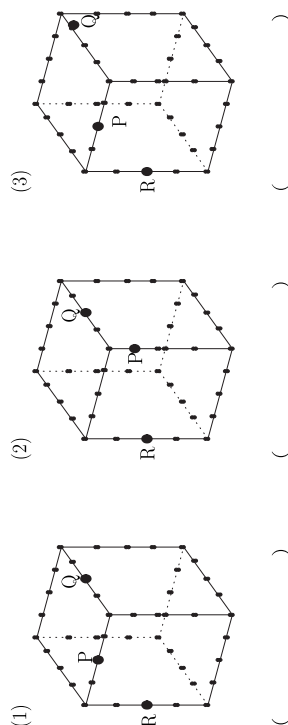
1. Open the "cube cross section.ggb" uploaded to Teams.
2. Select the menu "Point on Object" in GeoGebra, and draw a point P on the edge EF. (If it is not P, rename it.)
3. Next, take a point Q on the edge EH and a point R on the edge OE.
4. Check if you can move points P, Q, and R on the edge by dragging them.
5. Select the menu "Plane through 3 points" in GeoGebra and specify the points P, Q, and R.
6. Change the color of the plane to make it easier to see the shape of the cut plane. In the following operations, change the viewing direction as necessary.
7. Check the shape of the plane when point P is placed at vertex F, point Q is placed at vertex H, and point R is placed at vertex O.
8. For the figures (1) to (3) below, use the GeoGebra file you have created to examine the shape of the cut plane. Also, write the names of the shapes of the cut surfaces in parentheses below.
9. Save your GeoGebra file as a "... Download" file, rename the file to your number and name, and submit it via Teams.
10. Check out the other forms besides (1) through (3) below.



Your Name ()

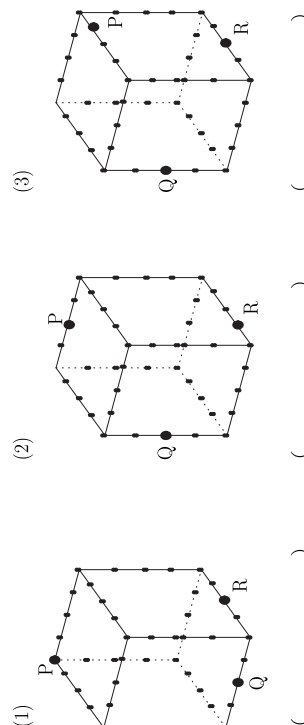
[Q2] Use GeoGebra to examine the cut plane of the cube below when cut by a plane passing through the three points P, Q, and R, and draw the result in the figure below. Note the symbols in the figure to indicate which lines are parallel and which lines are equal in length. Also, name the shapes of the cut surfaces in parentheses below.

Finally, save the GeoGebra file used in the discussion as a "Download in ... format," rename the file to your own name, and submit it via Teams.



[Q3] Use GeoGebra to examine the cut plane of the cube below when cut by a plane passing through the three points P, Q, and R, and draw the result in the figure below. Note the symbols on the figure to indicate which lines are parallel and which lines are equal in length. Also, name the shapes of the cut surfaces in parentheses below.

Submit the GeoGebra file used in the discussion, renaming the file to your own name, via Teams.



Please share your thoughts and impressions about using GeoGebra to examine a cube's cut surfaces.